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SU(4)/Z(2) Symmetry, Sextet Quarks, and a U(2) Gauge Theory

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ABSTRACT

Hadron symmetry schemes based on a sextet of quarks belonging to the 6 dimensional representation of SU(4)/Z(2) are explored with special focus on a model in which the electromagnetic and weak interactions can be unified into a spontaneously broken renormalizable $SU(2)\times U(1)$ gauge theory. Present data suggest the smaller gauge group U(2) in which case $M_W = 52.9$ GeV and $M_Z = 89.5$ GeV.

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The group SU(4) contains two discrete invariant subgroups: Z(2) and Z(4). Thus SU(4), SU(4)/Z(4) or SU(4)/Z(2) are possible hadron symmetry groups.[1] SU(4) corresponds to the various quartet extensions of the Gell-Mann - Zweig triplet model[2] and has been widely discussed.[3] SU(4)/Z(4) is the extension of the original "eightfold way" and does not involve quarks at all.[4] SU(4)/Z(2) corresponds to <u>sextet</u> quark models and is the extension of the Gell-Mann - Zweig triplet model that we investigate in this note.

We first explore the general features of all the SU(4)/Z(2) sextet quark model extensions and then present in detail one particularly attractive possibility which permits the unification of weak and electromagnetic interactions[5] into a spontaneously broken SU(2)× U(1), or perhaps U(2), renormalizable gauge theory. This specific sextet model with U(2) gauge symmetry agrees well with present data and offers many definitive experimental predictions.

Each SU(4) representation $(\Lambda_1, \Lambda_2, \Lambda_3)$ belongs to one of four classes depending on the value of $\Lambda_1 + 2\Lambda_2 + 3\Lambda_3$ (mod 4). Representations of SU(4)/Z(2) are only those of class 0 (bosons) and class 2 (fermions); their SU(3)×U(1) decompositions are given in Tables II and IV of Ref. 1.

The fundamental 6 contains the basic sextet of quarks, three of which are the Gell-Mann - Zweig triplet (p, n, λ) with charm[6] C = 0, while the other three (x, y, z) form an SU(3) antitriplet to which we assign C = -1.

The quark-antiquark meson multiplets are in the product[1]

$$6 \times 6 = 1 + 15 + 20 \tag{1}$$

We assume, as in SU(3), that the SU(4) breaking mixes the SU(2) and SU(3) singlets and SU(3) octets so that the familiar pseudoscalar and vector mesons are the usual combinations of p, n and λ quarks. The recently discovered neutral vector mesons[7] most naturally correspond to $^3S_1$ $q\bar{q}$ ground states. The $\psi(3.1)$ and $\psi'(3.7)$ are isoscalars, like ϕ and ω, while $\psi''(4.1)$ is an isovector, like ρ^0. This identification gives the crude quark mass estimates $m_p \simeq m_n \simeq 300$ MeV, $m_\lambda \simeq 450$ MeV, $m_z \simeq 1.9$ GeV and $m_x \simeq m_y \simeq 2.0$ GeV if we suppose the narrow $\psi(3.1)$ and $\psi'(3.7)$ lie below threshold and the broad $\psi''(4.1)$ is above threshold. From these estimates the $C = \pm 1$ $q\bar{q}$ meson states are expected to have masses in the range 2.0 to 2.5 GeV.

The three-quark baryon multiplets are in the product[1]

$$6 \times 6 \times 6 = 6 + 6 + 6 + 10 + \overline{10} + 50 + 64 + 64. \tag{2}$$

The $1/2^+$ baryons belong to the charm 0 octet in the 64 and the $3/2^+$ baryons belong to the charm 0 decouplet in the 50. From the above crude quark mass estimates the $C = -1$ qqq baryon states are expected to lie in the energy range 2 to 3 GeV.

Next we specify the various sextet quark models by identifying baryon number, isospin, charm (or hypercharm), electric charge and strangeness (or hypercharge) with the 1 + 15 generators F_i. In all models the baryon number

$$B = \sqrt{2/3}\, F_0 \;, \tag{3}$$

the three components of isospin are

$$I_i = F_i, \quad i = 1, 2, 3 \tag{4}$$

and the hypercharm, which we introduce for convenience, is

$$Y_c = C + \frac{3}{2} B = - \sqrt{\frac{3}{2}}\, F_{15} \, . \tag{5}$$

For the quarks p, n, λ, $Y_c = +1/2$ and for x, y, z, $Y_c = -1/2$.

The electric charge and hypercharge can be assigned in various ways, each of which specifies a different model. Requiring that p, n and λ have the usual values, the possibilities are

$$Q = F_3 + \sqrt{\frac{1}{3}}\, F_8 - \sqrt{\frac{3}{2}}\left(\frac{2}{3} - n_Q\right)(F_{15} + F_0) \tag{6}$$

and

$$Y = \frac{2}{\sqrt{3}}\, F_8 + \sqrt{\frac{3}{2}}\left(\frac{2}{3} - n_Y\right)(F_{15} + F_0) \tag{7}$$

where the integers n_Q and n_Y are restricted to 0 or 1 if $|\Delta Q| \le 1$ and $|\Delta Y| \le 1$. In any case

$$Q = I_3 + \frac{1}{2} Y + (1 - n_Q - \frac{1}{2} n_Y) C \, . \tag{8}$$

Next we discuss in some detail the specific sextet model: $n_Q = n_Y = 0$. The six quarks have charges[8] $Q_p = 2/3$, $Q_n = Q_\lambda = Q_x = Q_z = -1/3$, $Q_y = -4/3$ and hypercharges[9] $Y_z = 4/3$, $Y_p = Y_n = Y_x = Y_y = 1/3$, $Y_\lambda = -2/3$. This model has the attractive feature that it allows the unification of the weak and electromagnetic interactions into a spontaneously broken $SU(2)\times U(1)$ gauge theory that is renormalizable.[5] It is also supported by the present experimental data which, moreover, favor the smaller gauge group U(2). For U(2) there is only <u>one</u> gauge coupling constant, which is an extremely appealing possibility.

Under the SU(2) × U(1) gauge group, which we keep for the present, three of the six left-handed quarks p_L, $[(n+x)\cos\theta_c + (\lambda+z)\sin\theta_c]_L/\sqrt{2}$ and y_L form a Cabibbo-rotated triplet while $[-(n+x)\sin\theta_c + (\lambda+z)\cos\theta_c]_L/\sqrt{2}$, $(n-x)_L/\sqrt{2}$ and $(\lambda-z)/\sqrt{2}$ are singlets, all with weak hypercharge - 2/3. The six right-handed quarks are all singlets. The left-handed leptons $(\nu_e, \bar{e})_L$ and $(\nu_\mu, \mu^-)_L$ are both doublets with hypercharge -1 and the right-handed leptons e_R^- and μ_R^- are singlets with hypercharge -2, as usual. In addition, to eliminate the anomalous terms[10] in the divergences of the axial vector currents we postulate a new species of leptons:[11] δ^{++}, δ^+, ν_δ and δ^-. The left-handed deltons $(\delta^{++}, \delta^+, \nu_\delta, \delta^-)_L$ form an SU(2) quartet with weak hypercharge +1 and the right-handed deltons δ_R^{++}, δ_R^+, δ_R^- are singlets with weak hypercharges 4, 2 and -2, respectively. The delta neutrino ν_δ is a left-handed, two-component, massless neutrino.

In the resulting minimal gauge invariant interaction, the quark and lepton couplings to the gauge fields $W^\pm$, Z and A are

$$\begin{aligned}\mathscr{L} = \frac{1}{2\sqrt{2}}\, g \left[(J+L)_\lambda W_\lambda^- + \text{h.c.}\right] \\ - \frac{1}{2}(g^2+g'^2)^{1/2}(J^0+L^0)_\lambda Z_\lambda + e(J^{em}+L^{em})_\lambda A_\lambda\end{aligned} \tag{9}$$

where g and g′ are the SU(2) and U(1) coupling strengths, respectively, and $e = gg'/(g^2+g'^2)^{1/2}$ is the electric charge. The hadron currents are

$$\begin{aligned}J_\lambda &= \bar{p}\gamma_\lambda(1+\gamma_5)\left[(n+x)\cos\theta_c + (\lambda+z)\sin\theta_c\right] \\ &\quad + \left[(\bar{n}+\bar{x})\cos\theta_c + (\bar{\lambda}+\bar{z})\sin\theta_c\right]\gamma_\lambda(1+\gamma_5)y \\ &= \cos\theta_c(V-A)_\lambda^{1+i2} + \sin\theta_c(V-A)_\lambda^{4+i5} \\ &\quad - \sin\theta_c(V-A)_\lambda^{11+i12} + \cos\theta_c(V-A)_\lambda^{13+i14},\end{aligned} \tag{10}$$

$$J^0_\lambda = \bar{p}\gamma_\lambda(1+\gamma_5)p - \bar{y}\gamma_\lambda(1+\gamma_5)y - 2\sin^2\theta\, J^{em}_\lambda$$

$$= (V-A)^3_\lambda + \frac{1}{\sqrt{3}}(V-A)^8_\lambda - \sqrt{\frac{2}{3}}(V-A)^{15}_\lambda - 2\sin^2\theta\, J^{em}_\lambda , \tag{11}$$

$$J^{em}_\lambda = \frac{2}{3}\bar{p}\gamma_\lambda p - \frac{1}{3}(\bar{n}\gamma_\lambda n + \bar{\lambda}\gamma_\lambda\lambda + \bar{z}\gamma_\lambda z + \bar{x}\gamma_\lambda x) - \frac{4}{3}\bar{y}\gamma_\lambda y$$

$$= V^3_\lambda + \frac{1}{\sqrt{3}}V^8_\lambda - \sqrt{\frac{2}{3}}V^{15}_\lambda - \sqrt{\frac{2}{3}}V^0_\lambda , \tag{12}$$

and the lepton currents are

$$L_\lambda = \bar{\nu}_e\gamma_\lambda(1+\gamma_5)e + \bar{\nu}_\mu\gamma_\lambda(1+\gamma_5)\mu + \sqrt{3}\,\bar{\delta}^{++}\gamma_\lambda(1+\gamma_5)\delta^{+}$$

$$+ 2\bar{\delta}^{+}\gamma_\lambda(1+\gamma_5)\nu_\delta + \sqrt{3}\,\bar{\nu}_\delta\gamma_\lambda(1+\gamma_5)\delta^{-} , \tag{13}$$

$$L^0_\lambda = \frac{1}{2}\Big[\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\nu_e - \bar{e}\gamma_\lambda(1+\gamma_5)e + \bar{\nu}_\mu\gamma_\lambda(1+\gamma_5)\nu_\mu$$

$$- \bar{\mu}\gamma_\lambda(1+\gamma_5)\mu + 3\bar{\delta}^{++}\gamma_\lambda(1+\gamma_5)\delta^{++} + \bar{\delta}^{+}\gamma_\lambda(1+\gamma_5)\delta^{+}$$

$$- \bar{\nu}_\delta\gamma_\lambda(1+\gamma_5)\nu_\delta - 3\bar{\delta}^{-}\gamma_\lambda(1+\gamma_5)\delta^{-}\Big] - 2\sin^2\theta\, L^{em}_\lambda , \tag{14}$$

$$L^{em}_\lambda = -\bar{e}\gamma_\lambda e - \bar{\mu}\gamma_\lambda\mu + 2\bar{\delta}^{++}\gamma_\lambda\delta^{++} + \bar{\delta}^{+}\gamma_\lambda\delta^{+} - \bar{\delta}^{-}\gamma_\lambda\delta^{-} , \tag{15}$$

where the mixing angle $\sin\theta = g'/(g^2 + g'^2)^{1/2}$.

We assume interactions with scalar Higgs bosons[12] whose nonvanishing vacuum expectation value spontaneously breaks the $SU(2)\times U(1)$ gauge invariance; one doublet and three quartets are required. As a result of this spontaneous symmetry breaking all particles except the left-handed neutrinos ν_e, ν_μ and ν_δ and the photon γ acquire masses with the charged and neutral intermediate boson masses in the ratio

$$(M_W/M_Z)^2 = (7/10)\cos^2\theta \quad . \tag{16}$$

The Fermi constant G is related to g and M_W by

$$g^2/8M_W^2 = G/\sqrt{2} \tag{17}$$

as in the Weinberg-Salam model.[5] In the following discussion of the experimental consequences of this model we shall further assume that the gauge group is $U(2)$ rather than $SU(2)\times U(1)$, in which case $g = g' = \sqrt{2}e$,

$\sin^2\theta = 1/2$ and the gauge boson masses are determined to be

$$M_W^2 = \frac{e^2}{2\sqrt{2}\,G} = (52.9\ \text{GeV})^2 \tag{18}$$

and

$$M_Z^2 = (20/7)\,M_W^2 = (89.5\ \text{GeV})^2. \tag{19}$$

An additional interesting consequence of the U(2) gauge group is that the vector part of the hadronic neutral current becomes just the singlet baryon number current[13] and the leptonic neutral current becomes just one-half the electron plus muon minus delton number current.

Phenomenological features of the model include the following:

1) The weak neutral current is charm- and strangeness-conserving.

2) The charged current has both charm-conserving and charm-changing ($\Delta C = \Delta Q$) pieces and in both, the strangeness-changing part is suppressed by $\tan\theta_c$ relative to the strangeness-conserving part. The semileptonic strangeness-changing amplitudes obey $\Delta C = 0$, $\Delta S = \Delta Q = \pm 1$ and $\Delta S = -\Delta Q = -\Delta C = \pm 1$.

3) In the charged current neutrino reactions, charmed baryons can be singly produced by antineutrinos but not by neutrinos, e.g., $\bar{\nu}_\mu + \mu \rightarrow \mu^+ + B_c^0$. According to naive parton model calculations, neither the linear energy dependence nor the flat y-distribution of the neutrino cross section is altered by the production of charm. However, the antineutrino cross section increases from $\sigma^{\bar{\nu}N} = \frac{1}{3}\sigma^{\nu N}$ to equal $\sigma^{\nu N}$ well above charm threshold in the new scaling region. Although the $\bar{\nu}$ y-distribution in both scaling regions is of the form $(1-y)^2$, it could behave anomalously[14] in the transition region.

4) The neutral to charged current ratios for ν and $\bar{\nu}$ scattering on

isoscalar targets at energies in the two scaling regions are[15]

$$R^{\nu N} = 0.25 \tag{20}$$

and

$$R^{\bar{\nu} N} = \begin{cases} 0.44\,, & E < E_{TH} \\ 0.15\,, & E >> E_{TH} \end{cases} \quad . \tag{21}$$

For proton targets these ratios are[16]

$$R^{\nu p} = 0.49 \tag{22}$$

and

$$R^{\bar{\nu} p} = \begin{cases} 0.41\,, & E < E_{TH} \\ 0.16, & E >> E_{TH} \end{cases} \quad . \tag{23}$$

5) The leptonic neutral current cross sections are

$$\sigma(\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-) = 2\,\frac{G^2}{\pi}\, s\, (M_W/M_Z)^4 = 0.37\,\sigma_{V-A} \tag{24}$$

and

$$\sigma(\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-) = \left[1 + 3\,(M_W/M_Z)^4\right]\sigma_{V-A} = 1.37\,\sigma_{V-A} \tag{25}$$

in reasonable agreement with present data.[15]

6) The C = +1 meson antitriplet in the 15 contains an S = 0 isosinglet σ^+ and an S = -1 isodoublet (ξ^+, ξ^0) which is SU(4) favored to be produced in association with the C = -1 baryon triplet in the 64.

7) The nonleptonic interaction contains pieces transforming like components of the representations[1] 1, 15, 20 and 84 . The $\Delta C = +1$ amplitude transforms like 20 +84 and has $\Delta S = 0$, $\Delta S = -1$ and $\Delta S = -2$ parts which transform like the components of a U-spin triplet and are weighted by $\cos^2\theta_c$, $\sqrt{2}\sin\theta_c\cos\theta_c$ and $\sin^2\theta_c$, respectively. Thus the dominant charm-changing amplitude is strangeness conserving; it is also a V-spin singlet, which forbids $\xi^+ \to \bar{K}^0 + \pi^+$. The generalization of SU(3) octet domiance is the hypothesis that the 20 is enhanced over the 84 since the $\Delta I = 3/2$ pieces belong to the 84.

8) Asymptotically $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-) = 8$ according to the naive colored quark model calculation.[17]

9) The deltons δ^{++}, δ^{+}, ν_δ and δ^- conserve their own lepton number ℓ_δ as do the electronic and muonic leptons. Their masses are not restricted by the theory and presumably all are unstable except for ν_δ which is massless. They should be pair produced electromagnetically and could alter the ratio R.

A more complete discussion together with numerous additional experimental consequences will be presented in detail elsewhere.

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[8] For $n_Q = 1$, three quarks have $Q = 2/3$ and three have $Q = -1/3$ as in the SU(6) model of H. Harari, SLAC-Pub-1568. Within SU(4) no acceptable $SU(2) \times U(1)$ gauge theory can be constructed with these quark charges.

[9] For $n_Y = 1$, three quarks have $Y = 1/3$ and three have $Y = -2/3$. The resulting charm-changing weak current is predominantly strangeness-changing. While we choose to discuss the case $n_Y = 0$ in detail here, it is for experiment to decide, which, if either, is realized in nature.

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[15] These results for energies below charm threshold agree well with the data, cf. D. C. Cundy, Proceedings of the XVII International Conference on High Energy Physics, London (1974), p. IV - 131.

[16] The Weinberg-Salam model predicts $R^{\nu p} \simeq 0.29$ and $R^{\bar{\nu} p} \simeq 0.37$; cf. C. H. Albright, Phys. Rev. D8, 3162 (1973). Measurements of these ratios are in progress at Fermilab.

[17] Present data indicate $R \simeq 5$ to 6 as reported by C. C. Morehouse at the Washington APS Meeting, April 1975.

Table I. SU(3) × U(1) Decomposition Of Some Class 0 SU(4) Representations.

Representation $(\Lambda_1, \Lambda_2, \Lambda_3)$	Dimension $D(\Lambda)$	SU(3) × U(1) Representation C=2	C=1	C=0	C=-1	C=-2
(0, 0, 0)	1			1		
(1, 0, 1)	15		$\overline{3}$	1, 8	3	
(0, 2, 0)	20		6	8	$\overline{6}$	
(2, 1, 0)	45	3	$\overline{3}$, 6	8, 10	15′	
(0, 1, 2)	$\overline{45}$		$\overline{15}'$	8, $\overline{10}$	3, $\overline{6}$	$\overline{3}$
(2, 0, 2)	84	$\overline{6}$	$\overline{3}$, $\overline{15}'$	1, 8, 27	3, 15′	6

Table II. SU(3) × U(1) Decomposition of Some Class 2 SU(4) Representations. Hypercharm is given by $Y_c = C + (3/2)B$.

Representation $(\Lambda_1, \Lambda_2, \Lambda_3)$	Dimension $D(\Lambda)$	SU(3) × U(1) Decomposition $Y_c = 3/2$	$Y_c = 1/2$	$Y_c = -1/2$	$Y_c = -3/2$
(0, 1, 0)	6		3	$\overline{3}$	
(2, 0, 0)	10	1	3	6	
(0, 0, 2)	$\overline{10}$		$\overline{6}$	$\overline{3}$	1
(0, 3, 0)	50	10	15′	$\overline{15}'$	$\overline{10}$
(1, 1, 1)	64	8	3, $\overline{6}$, 15′	$\overline{3}$, 6, $\overline{15}'$	8